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Managing Interest Rate Option Risk

Managing Interest Rate Option Risk

Patrick Pancoast

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Option Portfolio Analysis

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Option Portfolio Analysis

Positions

Imagine you have the following positions on your (very simple) option book. All amounts are in thousands of DM:

Instrument	Position	Strike	Maturity	Face Value	Value
Cap #1	Short	7.50%	3	(DM15,000)	(DM141.60)
Cap #2	Long	8.50%	5	DM25,000	DM313.74
Floor #1	Long	6.50%	4	DM20,000	DM226.22
Floor #2	Short	6.25%	3	(DM50,000)	(DM325.05)
Pay Swaption	Short	7.50%	1×2	(DM25,000)	(DM180.57)
Rec. Swaption	Short	8.00%	2×3	(DM50,000)	(DM760.07)
Cash				DM1,000	DM1,000.00
TOTAL					**DM132.67**

An option pricing model tells you that the positions have the following sensitivities:

Instrument	Value	Δ	Γ	Vega	θ
Cap #1	(DM141.60)	(DM1.2197)	(DM0.0036)	(DM9.75)	DM0.14
Cap #2	DM313.74	DM2.5850	DM0.0074	DM34.07	(DM0.27)
Floor #1	DM226.22	(DM2.1518)	DM0.0075	DM19.02	(DM0.26)
Floor #2	(DM325.05)	DM3.6132	(DM0.0153)	(DM28.67)	DM0.55
Pay Swaption	(DM180.57)	(DM1.9387)	(DM0.0059)	(DM11.16)	DM0.26
Rec. Swaption	(DM760.07)	DM5.0098	(DM0.0121)	(DM44.47)	DM0.46
Cash	DM1,000.00	DM0.0000	DM0.0000	DM0.00	DM0.00
TOTAL	**DM132.67**	**DM5.90**	**(DM0.02)**	**(DM40.97)**	**DM0.88**

The delta of the above position is not great. It tells us that the portfolio will lose about 5% of its value for each basis point move down in the yield curve. Clearly for large moves of interest rates this is risky.

Gamma is also working against us. As rates move lower, the loss from the delta will grow faster and faster. It will accelerate at DM0.02 per 0.01% of change in rates.

The vega position exposes us to changing volatility. We are short volatility, so any increase in volatility will cost us money at a rate of DM40.97 per 1% change in volatility.

Finally, since we have mostly sold options, our liability for them decays at about DM0.88 per day assuming nothing else changes.

Mapping Cash Flows

The process of mapping the portfolio allows us to measure the exposure we have to changes in the underlying market input rates.

Liquid Market Rates

Exchange-traded money market futures (contracts on 3-month LIBOR such as the Eurodollar, Euromark, etc.) are the most liquid market instruments the trader can use to adjust the value sensitivity of the portfolio.

The DM yield curve can thus be defined by using the first 6 contracts of Euromark futures traded on LIFFE followed by the interest rate swaps out to 10 years.

To manage a portfolio, it is vital to know the sensitivity of the portfolio to movements in the liquid instruments which can be used to adjust it. To change the rate sensitivity, the book manager must trade, and he will always prefer to trade the most liquid instruments. Therefore, we want to price and measure sensitivity in terms of the same liquid instruments.

What is required is a table of sensitivities:

Instrument	Interest Rate Portfolio Sensitivity
Mar 95 Future	?
Jun 95 Future	?
Sep 95 Future	?
Dec 95 Future	?
Mar 96 Future	?
Jun 96 Future	?
2-Year Swap Rate	?
3-Year Swap Rate	?
4-Year Swap Rate	?
5-Year Swap Rate	?

If the above table is defined, a position in the most liquid instrument can be used to offset the risk.

As such the trader can pick and choose at what points along the yield curve to remove exposure by trading in those particular instruments.

In addition, new customer business (swaps, options, etc.) can be analyzed by the trader and the effect on the risk of the current book can be easily identified.

A trader has two ways to manage the position and risk of the portfolio:

1. Use exchange traded futures or interbank derivatives (swaps, caps, floors, etc.)
2. Execute customer transactions

Exchange traded futures and interbank trading both have a cost: For futures it is the commission cost and margin calls. For trading in the interbank market, the cost is the bid/offer spread.

If the trader can execute a deal with a customer that moves the risk of the book in a desirable direction the pricing may be slightly better (for the book) than the market quote.

The trader can afford to give better pricing because the alternative is to execute a transaction to hedge certain exposures which will cost money. Some of that savings can be passed onto the customer.

If a customer trade would change the sensitivity of the portfolio to an undesirable level, the trader may quote a price worse than the market.

If the customer trade is executed, the trader will have to go out and pay to execute transactions which will return the book to the desirable position.

Delta

We can delta hedge the portfolio by taking positions to offset the portfolio's base exposure to 0.01% changes in the market input rates. First we have to measure it.

Following is the change in the price of the first cap when each of the input rates is changed by 1 basis point up and down.

Cap #1		Rates Up 0.01%	Rates Down 0.01%	Average Change
Input Rates				
Mar-95	94.83	DM0.18	(DM0.18)	DM0.18
Jun-95	94.49	DM0.16	(DM0.16)	DM0.16
Sep-95	94.10	DM0.15	(DM0.16)	DM0.16
Dec-95	93.70	DM0.11	(DM0.11)	DM0.11
Mar-96	93.36	DM0.05	(DM0.05)	DM0.05
Jun-96	93.08	DM0.03	(DM0.03)	DM0.03
2 Yr	6.4450%	DM0.28	(DM0.32)	DM0.30
3 Yr	6.8450%	(DM2.16)	DM2.13	(DM2.15)
4 Yr	7.0650%	DM0.00	DM0.00	DM0.00
5 Yr	7.2150%	DM0.00	DM0.00	DM0.00

If we measure the sensitivity of each instrument to 0.01% changes in the input rates, we obtain the following table of sensitivities for our portfolio:

Input Rates		Portfolio Sensitivities
Mar-95	94.83	DM0.1738
Jun-95	94.49	DM0.3972
Sep-95	94.10	DM0.5664
Dec-95	93.70	DM0.3128
Mar-96	93.36	DM0.0267
Jun-96	93.08	DM0.0147
2 Yr	6.4450%	(DM4.1043)
3 Yr	6.8450%	(DM2.7138)
4 Yr	7.0650%	(DM1.9659)
5 Yr	7.2150%	DM13.5947

These represent the delta of the portfolio to 0.01% changes in the market rates used to construct the pricing curve. Positive numbers mean a gain if the market rate rises.

If the price of the March 1995 futures contract moves down by 1 tick, i.e. if the rate implied by it (5.17%) moves up by 0.01%, the portfolio will gain DM0.1738.

Most of the delta exposure is to the 5-year swap rate. If it moves up by 0.01%, the portfolio will gain DM13.5947.

To "delta hedge" the portfolio, we need to know the sensitivities of each of the underlying input instruments to a one basis point change, too.

The futures are easy. Each basis point is worth the tick value, which is DM25. For the par swaps we measure the sensitivity by revaluing each par swap for a 0.01% change in the par swap rates. The numbers in both cases assume a unit value of DM1 mio.

Input Rates		Market Equivalents
Mar-95	94.83	(DM25)
Jun-95	94.49	(DM25)
Sep-95	94.10	(DM25)
Dec-95	93.70	(DM25)
Mar-96	93.36	(DM25)
Jun-96	93.08	(DM25)
2 Yr	6.4450%	(DM182.77)
3 Yr	6.8450%	(DM264.65)
4 Yr	7.0650%	(DM340.59)
5 Yr	7.2150%	(DM410.94)

Both futures and swaps lose value as rates rise. For the futures, this means that the equivalent position is long a futures contract. This position loses value as rates rise.

For the swaps, this is equivalent to receiving fixed. The receiver of fixed is similar to someone long a cash bond: as rates rise the position loses value.

By comparing the basis point sensitivity of the portfolio to that of the input instruments, we can describe how to delta hedge the portfolio:

Input Rates		Portfolio Sensitivities	Market Equivalents	Delta Hedge
Mar-95	94.83	DM0.1738	(DM25)	7.0
Jun-95	94.49	DM0.3972	(DM25)	15.9
Sep-95	94.10	DM0.5664	(DM25)	22.7
Dec-95	93.70	DM0.3128	(DM25)	12.5
Mar-96	93.36	DM0.0267	(DM25)	1.1
Jun-96	93.08	DM0.0147	(DM25)	0.6
2 Yr	6.4450%	(DM4.1043)	(DM182.77)	Pay(DM22.457)
3 Yr	6.8450%	(DM2.7138)	(DM264.65)	Pay(DM10.254)
4 Yr	7.0650%	(DM1.9659)	(DM340.59)	Pay(DM5.772)
5 Yr	7.2150%	DM13.5947	(DM410.94)	RecDM33.082

By buying the indicated number of futures contracts and either receiving or (paying) fixed in the indicated amounts (in DM mio) we can delta hedge the portfolio.

The exposures to the futures contracts will lose money if rates fall (if prices rise). To hedge this we need long positions in the futures.

The calculation of the 5-year par swap hedge is as follows:

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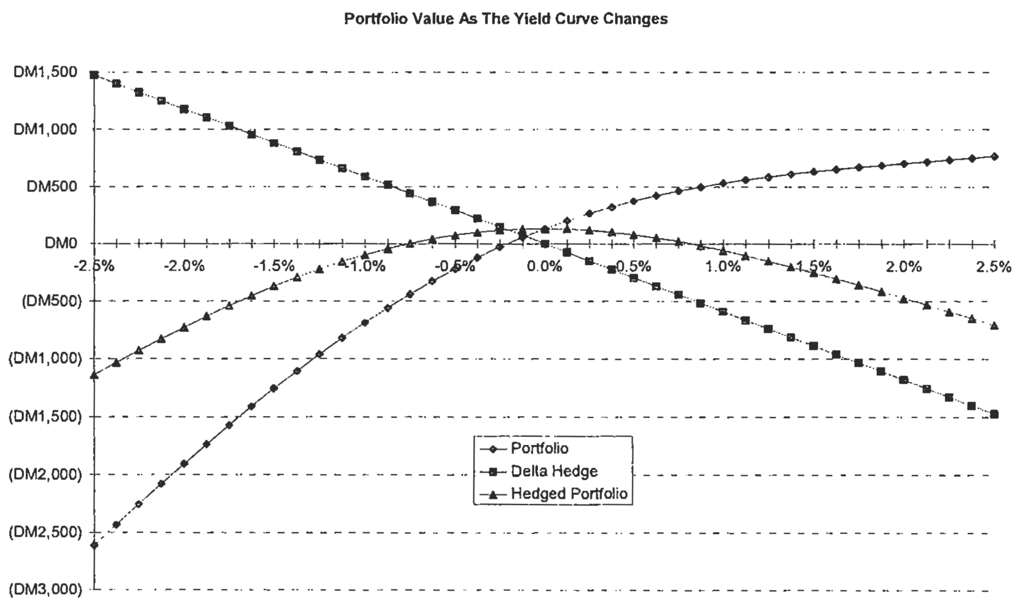
$$\frac{13.5947 \times 1{,}000 \times 1{,}000{,}000}{410.94} = 33{,}082{,}000$$

To hedge the 5-year par swap exposure, we need to receive fixed on a 5-year par swap of face value DM33.082 mio.

The 5-year swap equivalent risk is due to the 2×3 receiver swaption we sold and the 5-year cap #2 we own, which are both tied to changes in the 5-year par swap rate.

Gamma

The delta hedge is only valid for small changes in the yield curve, as can be seen in the following graph.



After delta-hedging our portfolio, we are left with a position which will lose money if rates move either up or down.

This is the problem of gamma. The option portfolio has gamma, as can be seen in the graph of its value at a range of possible interest rate change levels. As rates fall, the portfolio loses value at a faster and faster pace.

The futures and swap positions used to create the delta hedge have very little gamma. The delta hedge serves to hedge part of the risk for small movements in interest rates. But the delta hedge falls further and further behind if rates continue to fall.

The effectiveness of the delta hedge for small changes in interest rates can be seen by looking at the middle of the graph above:

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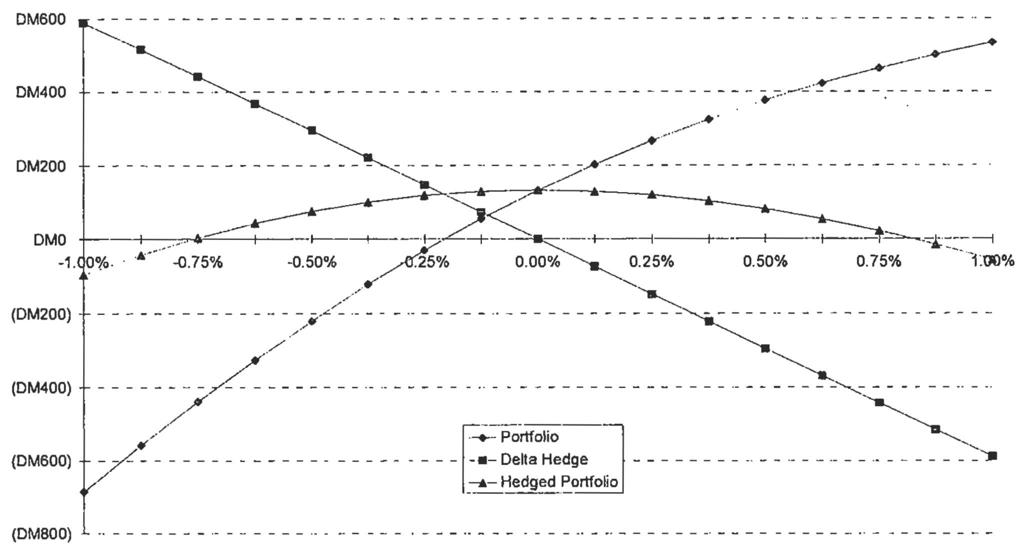
Portfolio Value as the Yield Curve Changes



For small movements of less than 0.75% in either direction, the delta hedge preserves part of the profit in the option portfolio.

But as rates continue to move, the delta hedge has to be rebalanced or the losses will grow.

The gamma is greatest on the 1×2 payer swaption we sold, and it makes a good example of the gamma risk. To delta hedge it, we need to pay fixed on par swaps.

As rates move down, the owner is less and less likely to exercise, and we need to unwind the hedge, in effect receiving fixed at lower and lower rates.

As rates move up, the owner is more and more likely to exercise, and we need to have more and more hedge, paying fixed at higher and higher rates.

Whichever way rates move, they are moving against us.

Vega

We can map the vega of the portfolio to the underlying cap and swaption volatilities used to price the caps, floors and swaptions. In effect we are creating a "vega map" in terms of the instruments which might be used to hedge it.

Instrument		Cap #1	Cap #2	Floor #1	Floor #2	Pay Swaption	Rec Swaption	TOTAL
Face Value		(DM15,000)	DM25,000	DM20,000	(DM50,000)	(DM25,000)	(DM50,000)	
Value		(DM141.60)	DM313.74	DM226.22	(DM325.05)	(DM180.57)	(DM760.07)	(DM867.33)
Cap Volatility								
1 Yr	19.50%	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00
2 Yr	17.80%	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00
3 Yr	17.20%	(DM9.80)	DM0.00	DM0.00	(DM28.97)	DM0.00	DM0.00	(DM38.76)
4 Yr	16.20%	DM0.00	DM0.00	DM19.182	DM0.00	DM0.00	DM0.00	DM19.18
5 Yr	15.50%	DM0.00	DM34.39	DM0.00	DM0.00	DM0.00	DM0.00	DM34.39
Swaption Volatility								
1×2	17.03%					(DM11.16)	DM0.00	(DM11.16)
1×4	15.64%					DM0.00	DM0.00	DM0.00
2×3	14.98%					DM0.00	(DM44.47)	(DM44.47)

In the map above, the vegas are mapped to the term volatilities of the various caps and floors, and swaptions. This is done by varying each volatility input by 1% and recalculating the option prices. Done in this manner, each instrument is sensitive to only one market volatility input.

If we assume that the term structure of volatility shifts up and down in parallel fashion, we can try and hedge the entire vega risk with a single purchased option. We can also hedge the vega of the portfolio to each cap and swaption volatility by using ATM caps, floors and swaption to hedge each bucket separately.

There are other ways to map vega, too. One common approach is to map to the implied forward forward volatilities, which would map part of each cap and floor to various forward forward volatilities.

It is also feasible to tie the swaptions to the map of forward forward volatilities, or even to map the whole portfolio to ATM swaption volatility, to use purchased swaptions to hedge short cap and floor vega.

Theta

The theta of the portfolio is DM0.88. This means that it will gainDM0.88 per day, assuming all else is constant. This will of course gather speed as the various instruments move nearer to expiry.

Adding a Swaption

Now let us imagine we have a client who wishes to purchase DM100 mio of 1×4 payer swaption struck at 7.75%. The 1×4 forward swap rate is currently 7.6482% and the appropriate volatility is 15.64%.

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According to the option pricing model, the value of this swaption is DM1,292.19. We agree to charge the client this price and he buys the swaption. What does this do to the portfolio we already had in place?

New Position and Delta Hedge

The portfolio now consists of the following:

Instrument	Position	Strike	Maturity	Face Value	Value
Cap #1	Short	7.50%	3	(DM15,000)	(DM141.60)
Cap #2	Long	8.50%	5	DM25,000	DM313.74
Floor #1	Long	6.50%	4	DM20,000	DM226.22
Floor #2	Short	6.25%	3	(DM50,000)	(DM325.05)
Pay Swaption	Short	7.50%	1×2	(DM25,000)	(DM180.57)
Rec. Swaption	Short	8.00%	2×3	(DM50,000)	(DM760.07)
Pay Swaption	Short	7.75%	1×4	(DM100,000)	(DM1,292.19)
Cash				DM2,523	DM2,292.19
TOTAL					**DM132.67**

An option pricing model tells you that the positions have the following sensitivities:

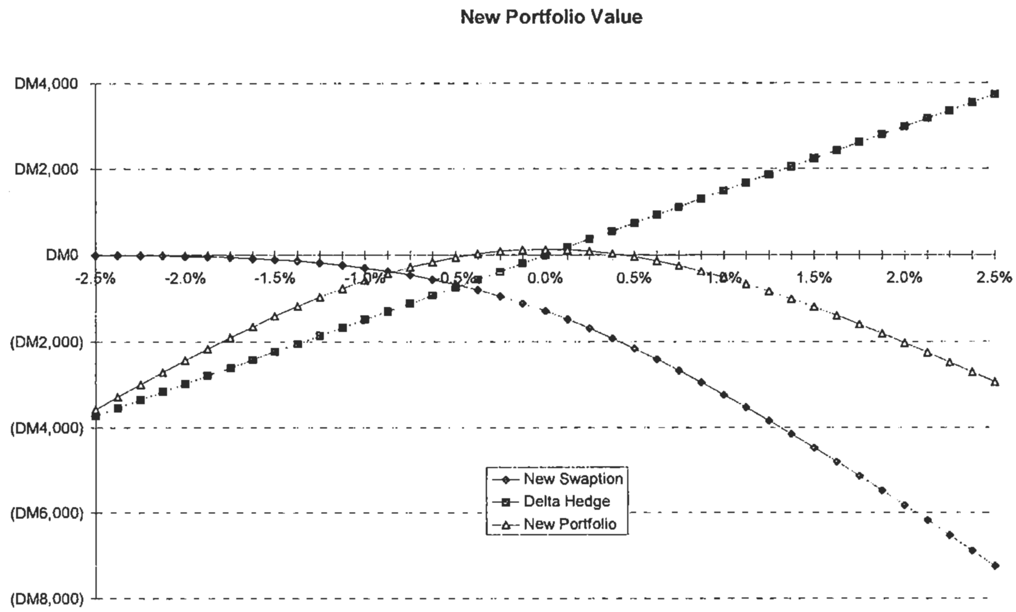
Instrument	Value	Δ	Γ	Vega	θ
Cap #1	(DM141.60)	(DM1.2197)	(DM0.0036)	(DM9.75)	DM0.14
Cap #2	DM313.74	DM2.5850	DM0.0074	DM34.07	(DM0.27)
Floor #1	DM226.22	(DM2.1518)	DM0.0075	DM19.02	(DM0.26)
Floor #2	(DM325.05)	DM3.6132	(DM0.0153)	(DM28.67)	DM0.55
Pay Swaption	(DM180.57)	(DM1.9387)	(DM0.0059)	(DM11.16)	DM0.26
Rec. Swaption	(DM760.07)	DM5.0098	(DM0.0121)	(DM44.47)	DM0.46
Pay Swaption	(DM1,292.19)	(DM14.8823)	(DM0.0498)	(DM91.49)	DM1.95
Cash	DM1,000.00	DM0.0000	DM0.0000	DM0.00	DM0.00
TOTAL	**DM132.67**	**(DM8.9845)**	**(DM0.0717)**	**(DM132.46)**	**DM2.83**

The delta has shifted from positive to negative, which means the overall position is now equivalent to paying fixed on a lot of swaps. The gamma has jumped, and will be working even more against us if rates move much at all.

The delta map now looks like this:

Input Rates		New Swaption	Delta Hedge	New Portfolio	New Hedge
Mar-95	94.83	DM1.34	53.4	DM1.5100	60.4
Jun-95	94.49	DM1.24	49.6	DM1.6382	65.5
Sep-95	94.10	DM1.24	49.6	DM1.8061	72.2
Dec-95	93.70	DM0.66	26.3	DM0.9701	38.8
Mar-96	93.36	DM0.00	0.0	DM0.0267	1.1
Jun-96	93.08	DM0.00	0.0	DM0.0147	0.6
2 Yr	6.4450%	DM0.03	DM0.158	(DM4.0755)	(DM22.299)
3 Yr	6.8450%	DM0.04	DM0.167	(DM2.6697)	(DM10.087)
4 Yr	7.0650%	DM0.06	DM0.178	(DM1.9053)	(DM5.594)
5 Yr	7.2150%	(DM19.28)	(DM46.920)	(DM5.6865)	(DM13.838)

Assuming we adjust our delta hedge to the amounts in the far right column, our portfolio now faces interest rate exposure even greater than before:



The new swaption will cost us money if rates rise, as its owner will be more and more likely to exercise into more and more profit (for him).

The delta hedge of the new swaption is paying fixed on DM47 mio of 5-year swap offset by buying large amounts of the four nearby futures. The combined effect is to leave us with a large amount of gamma.

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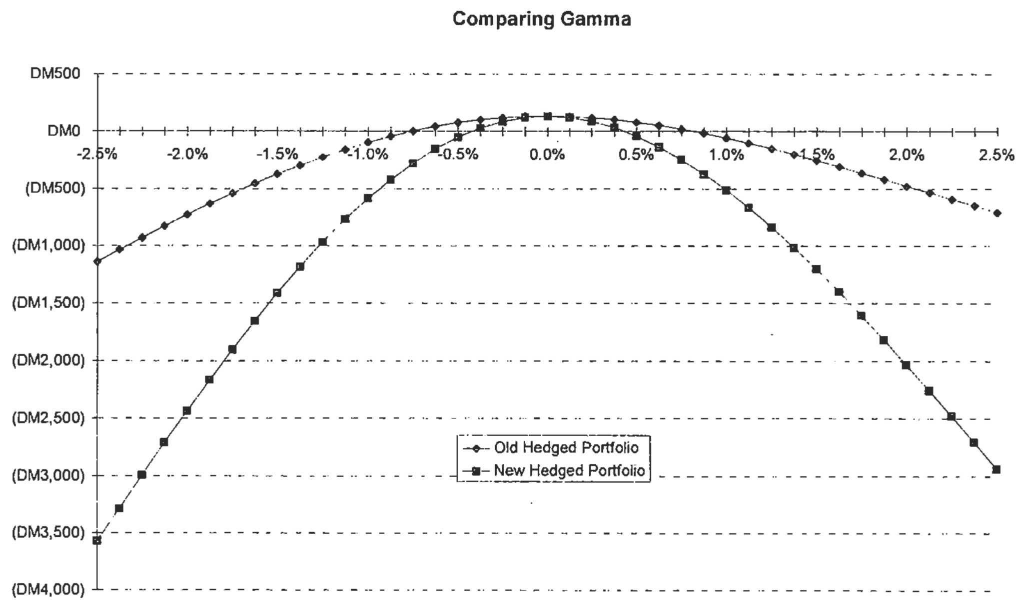
Gamma Effect

The change in the gamma of the portfolio is from DM0.02 per basis point of rate movement to DM0.07 per basis point of rate movement.

Since we sold more options, gamma works against us harder than ever. The change is easy to see when we compare the delta-hedged portfolio value across a range of interest rate changes before and after the new swaption:



For option books consisting mostly of sold options, gamma is highly problematic.

There is only one way to hedge it: buy options sensitive to the same interest rate changes.

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Hedging Vega

In order to hedge this option book more effectively, we need to buy options. Another approach used by many market participants is to begin with the vega, offsetting that and then hedging the residual delta risk.

This approach has one major positive: it also usually offsets the gamma risk.

It has one big negative, too: it means buying large quantities of options from the market. If client business is strong on both sides, i.e. clients are both buying and selling options to the bank, bid-offer spreads can mean nice profits with far less risk. If clients are mostly buying options from the bank, and the bank has to buy its hedges from the market, the profit of the client business will be far smaller.

In this case, let us imagine we wish to buy the same 1×4 payer swaptions to hedge the vega. How much do we need to buy?

The vega of the 1×4 payer swaption was DM91.49 per DM100,000. As a percent of face value, vega is thus: $\frac{91.49}{100,000} = 0.09149\%$.

The vega in the new portfolio is (DM132.46). To offset this much vega, we need to buy DM144,783 worth of the 1×4 payer swaption:

$$\text{Hedge} = \frac{132.46}{0.09149\%} = 144,783$$

At a price of 1.29219%, this will cost DM1,871. The portfolio now consists of:

Instrument	Position	Strike	Maturity	Face Value	Value
Cap #1	Short	7.50%	3	(DM15,000)	(DM141.60)
Cap #2	Long	8.50%	5	DM25,000	DM313.74
Floor #1	Long	6.50%	4	DM20,000	DM226.22
Floor #2	Short	6.25%	3	(DM50,000)	(DM325.05)
Pay Swaption	Short	7.50%	1×2	(DM25,000)	(DM180.57)
Rec. Swaption	Short	8.00%	2×3	(DM50,000)	(DM760.07)
Pay Swaption	Long	7.75%	1×4	DM44,783	DM578.68
Cash				DM421	DM421.32
TOTAL					**DM132.67**

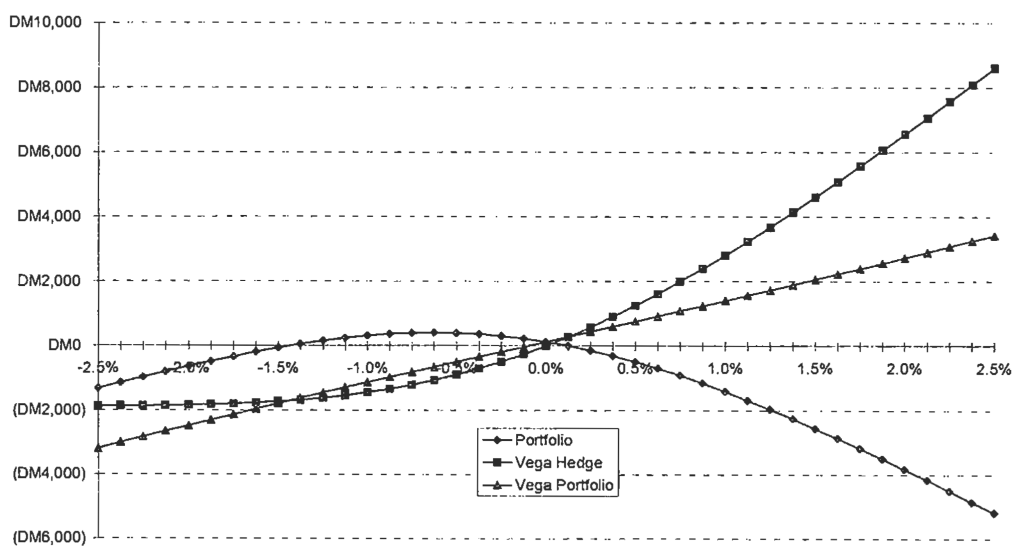
An option pricing model tells you that the positions have the following sensitivities:

Instrument	Value	Δ	Γ	Vega	θ
Cap #1	(DM141.60)	(DM1.2197)	(DM0.0036)	(DM9.75)	DM0.14
Cap #2	DM313.74	DM2.5850	DM0.0074	DM34.07	(DM0.27)
Floor #1	DM226.22	(DM2.1518)	DM0.0075	DM19.02	(DM0.26)
Floor #2	(DM325.05)	DM3.6132	(DM0.0153)	(DM28.67)	DM0.55
Pay Swaption	(DM180.57)	(DM1.9387)	(DM0.0059)	(DM11.16)	DM0.26
Rec. Swaption	(DM760.07)	DM5.0098	(DM0.0121)	(DM44.47)	DM0.46
Pay Swaption	DM578.68	DM6.6647	DM0.0223	DM40.97	(DM0.87)
Cash	DM421.32	DM0.0000	DM0.0000	DM0.00	DM0.00
TOTAL	**DM132.67**	**DM12.5626**	**DM0.0004**	**DM0.00**	**DM0.01**

This is a very interesting result. We have a lot of delta risk, but vega is down to zero, and the gamma is nearly all offset, as is theta.

This is shown on the graph following:

Hedging Vega by Buying Swaptions



All that remains is to hedge the delta and see where we stand for various interest rate movements.

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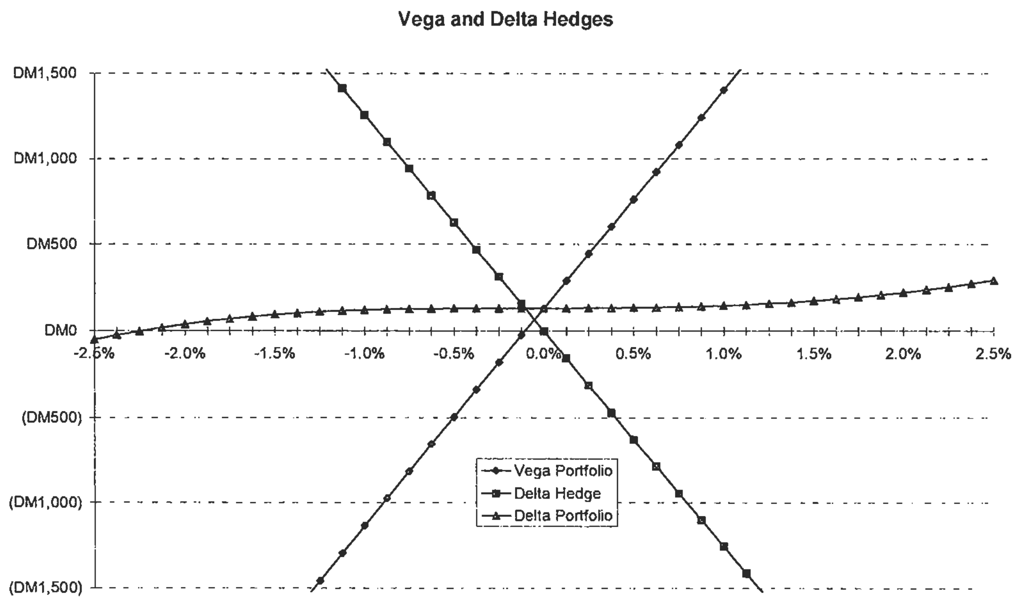
Hedging Delta

To hedge the delta, we need to map the delta to the market input instruments once more.

Input Rates		Previous Portfolio	New Vega Swaption	New Portfolio	New Delta Hedge
Mar-95	94.83	DM0.1738	(DM1.9346)	(DM1.7608)	-70.4
Jun-95	94.49	DM0.3972	(DM1.7967)	(DM1.3994)	-56.0
Sep-95	94.10	DM0.5664	(DM1.7949)	(DM1.2285)	-49.1
Dec-95	93.70	DM0.3128	(DM0.9516)	(DM0.6388)	-25.6
Mar-96	93.36	DM0.0267	DM0.0000	DM0.0267	1.1
Jun-96	93.08	DM0.0147	DM0.0000	DM0.0147	0.6
2 Yr	6.4450%	(DM4.1043)	(DM0.0418)	(DM4.1461)	(DM22.685)
3 Yr	6.8450%	(DM2.7138)	(DM0.0638)	(DM2.7776)	(DM10.495)
4 Yr	7.0650%	(DM1.9659)	(DM0.0878)	(DM2.0537)	(DM6.030)
5 Yr	7.2150%	DM13.5947	DM27.9159	DM41.5105	DM101.014

Assuming that is done and we adjust the delta hedge once more, we can measure the overall position across a range of interest rates:



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Pancoast Seminars

Financial Product Education,
Analytics, & Consulting
Capital Markets, Derivative Products, &
Asset Management

Gamma

If we are willing to invest in the large payer swaption, gamma is hedged while we are hedging volatility.

Hedging delta only is better than not hedging at all. The same is true for hedging vega.

Hedging delta alone often increases gamma and vega.

Hedging vega and delta together may be a far more efficient way to protect the book's profits.

A well-constructed hedge of vega and gamma together is normally far more stable than hedging either one separately as both volatility and interest rate move.

No hedge is permanent. New deals are being booked all the time. Liquidity of futures, swaps, OTC options and futures options (where available), combined with a diversity of client deals leads to greater profitability from the option business.

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